BRAINYCHALKS

Minor Assessment - 26

Maximum Marks: 60 **Time Allowed: 2½ Hours**

SECTION A – MULTIPLE-CHOICE QUESTIONS (6 × 1 = 6 marks)

1. The function $f(x) = x^2 - 4x + 6$ is increasing in the interval:
   (a) (0, 2)
   (b) $(-\infty, 2]$
   (c) [1, 2]
   (d) $[2, \infty)$
2. If $f(x) = a(x - \cos x)$ is strictly decreasing in R, then a belongs to:
   (a) {0}
   (b) $(0, \infty)$
   (c) $(-\infty, 0)$
   (d) $(-\infty, \infty)$
3. If $y = a\cos(\log x) + b\sin(\log x)$, then $x^2y'' + xy' + y$ is equal to:
   (A) 0
   (B) y
   (C) xy
   (D) 1
4. **2.**If $y = e^{-x}$, then $\frac{d^2y}{dx^2}$ is:
   (A) −y
   (B) y
   (C) x
   (D) −x
5. The set of points where $f(x) = x + |x|$ is differentiable is:
   (A) $(0, \infty)$
   (B) $(-\infty, 0)$
   (C) $(-\infty, 0) \cup (0, \infty)$
   (D) $\mathbb{R}$
6. If $f(x) = \cos^{-1}(\sqrt{x})$, $0 < x < 1$, then f'(x) is
   (A) $\frac{-1}{2\sqrt{(x(1-x)}}$
   (B) $\frac{1}{\sqrt{(1-x)}}$
   (C) $\frac{1}{2\sqrt{x(1-x)}}$
   (D) $\frac{-1}{\sqrt{1-x}}$
7. For $y = (\sin^{-1}x)^2$, the first derivative dy/dx is:
   (A) $\frac{2sin^{-1}x}{\sqrt{1-x^2}}$
   (B) $\frac{sin^{-1}x}{\sqrt{1-x^2}}$
   (C) $\frac{2x}{(1-x^2)}$
   (D) $\frac{1}{(1-x^2)}$
8. The value of $\int_0^a f(a-x)dx$, after the substitution t = a−x, is: **[1 mark]**
   (A) $\int_0^a f(t)dt$
   (B) $-\int_0^a f(t)dt$
   (C) f(a)−f(0)
   (D) 0

SECTION B – VERY SHORT ANSWER QUESTIONS (5 × 2 = 10 marks)

7. If $x = a\cos t$ and $y = 2at$, find $\frac{d^2y}{dx^2}$

8. If $y = (\tan^{-1}x)^2$, show that $(x^2 + 1)^2\left(\frac{d^2y}{dx^2}\right) + 2x(x^2 + 1)\left(\frac{dy}{dx}\right) = 2$.

9. Evaluate: $\int \frac{(x+5)e^x}{(x+6)^2}\,dx$.

10. Evaluate: $\int_0^1 x^2(1-x)^n dx$.

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11. If y = sin(sin x), write the differential equation satisfied by y: $\frac{d^2y}{dx^2} + \tan x\left(\frac{dy}{dx}\right) + y\cos^2 x = 0$. Verify by differentiation.

SECTION C – SHORT ANSWER QUESTIONS (6 × 3 = 18 marks)

12. If $y = (\sin^{-1}x)^2$, prove that $(1 - x^2)^2\left(\frac{d^2y}{dx^2}\right) - x\left(\frac{dy}{dx}\right) - 2 = 0$.

13. If y = tan x + sec x, prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1 - \sin x)^2}$.

14. If x = a cos θ and y = b sin θ, find $\frac{d^2y}{dx^2}$.

15. Evaluate: $\int_0^a f(x)dx = \int_0^a f(a - x)dx$, and use this property to simplify $\int_0^a \frac{f(x)}{f(x)+f(a-x)}dx$.

16. Evaluate: $\int \frac{(\sin x + \cos x)}{(16 + 9\sin 2x)}dx$.

17. If $y = (\log x)^2$, prove that $x^2y'' + xy' = 2$.

SECTION D – LONG ANSWER QUESTIONS (4 × 5 = 20 marks)

18. Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ with respect to $\cos^{-1}\left(2x\sqrt{1 - x^2}\right)$, where $x \in \left(\frac{1}{\sqrt{2}}, 1\right)$.

19. If y = a cos(log x) + b sin(log x), prove that $x^2\left(\frac{d^2y}{dx^2}\right) + x\frac{dy}{dx} = 0$.

20. If $x = a\left(\cos\theta + \log\tan\left(\frac{\theta}{2}\right)\right)$ and y = sin θ, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

21. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{x}{\sin x + \cos x}dx$.

SECTION E – CASE-BASED QUESTION (1 × 4 = 4 marks)

22. Case Study: A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat-bottomed tank.



A tap is connected to a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uniform rate of 2 cm³/s. The semi-vertical angle of the conical tank is 45°.

On the basis of the given information, answer the following questions:

(i) Find the volume of water in the tank in terms of its radius r.

(ii) Find the rate of change of radius at an instant when $r = 2\sqrt{2}$ cm.

(iii) (a) Find the rate at which the wet surface of the conical tank is decreasing at an instant when radius $r = 2\sqrt{2}$ cm.

OR

(iii) (b) Find the rate of change of height h at an instant when the slant height is 4 cm. (2023)

— END OF QUESTION PAPER —