BRAINYCHALKS

Institute for CBSE | JEE | NEET | OLYMPIAD

MONTHLY ASSESSMENT - I

Marks- 40 **SUBJECT – MATHEMATICS** **Time – 1hr 30min**

General Instructions

Read the following instructions very carefully and strictly follow them :

(i) This question paper contains 17 questions. All questions are compulsory.

(ii) This question paper comprises five Sections A, B, C, D and E .

(iii) Section A Questions no. 1 to 7 are MCQs of 1 mark each.

(iv) Section B Questions no. 8 to 10 are very short answer type questions, carrying 2 marks each. Answer to each question should not exceed 40 words.

(v) Section C Questions no. 11 to 14 are short answer type questions, carrying 3 marks each. Answer to each question should not exceed 60 words.

(vi) Section D Questions no. 15 to 17 are long answer type questions, carrying 5 marks each. Answer to each question should not exceed 120 words.

(vii) There is no overall choice in the question paper. However, an internal choice has been provided in few questions. Only one of the choices in such questions has to be attempted.

(viii) In addition to this, separate instructions are given with each section and question, wherever necessary.

SECTION A $(7\times 1 = 07)$

1. If f(x) = $\begin{cases} \frac{\log(1+ax)+\log(1-bx)}{x}, \text{for } x \neq 0 \\ k \text{ for } x = 0 \end{cases}$ is continuous at x=0, then the value of k is:
   a) a
   b) a+b
   c) a-b
   d) b
2. The number of points of discontinuity of
   F(x)=$\begin{cases} |x| + 3, \text{if } x \leq -3 \\ -2x, \text{if } -3 < x < 3 \\ 6x + 2, \text{if } x \geq 3 \end{cases}$
   a) 0
   b) 1
   c) 2
   d) Infinite
3. The function f(x)=[x] denotes the greatest integer less than or equal to x, is continuous at
   a) X=1
   b) X=1.5
   c) X=-2
   d) X=4

4. If the function $f(x) = \begin{cases} 3x - 8, & x \leq 5 \\ 2k, & x > 5 \end{cases}$ is continuous , then the value of k is

a) 2/7
b) 7/2
c) 3/7
d) 4/7

5. The function $f(x) = \begin{cases} x^2, & x < 1 \\ 2 - x, & x \geq 1 \end{cases}$ is

a) Not differentiable at x=1
b) Differentiable at x=1
c) Not continuous at x=1
d) Neither continuous nor differentiable at x=1

6. If f(x)=|x| + |x-1|, then which of the following is correct?

a) F(x) is both continuous and differentiable at x=0 and x=1
b) F(x) is differentiable but not continuous at x=0 and x=1
c) F(x) is continuous but not differentiable at x=0 and x=1
d) F(x) is neither continuous nor differentiable at x=0 and x=1

7. If $\tan^{-1}\left(x^2 + y^{2)}\right) = a$ where 'a' is a constant, then $\frac{dy}{dx}$ is:

a) $\frac{x}{y}$
b) $-\frac{x}{y}$
c) $\frac{a}{x}$
d) $\frac{a}{y}$

SECTION B $(3\times 2 = 06)$

8. If $x = a\sec\theta, y = b\tan\theta$, then find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$
9. Find the differential of $\sin^2 x$ w.r.t $e^{\cos x}$
10. If $x = a\cos\theta; y = b\sin\theta$ then find $\frac{d^2y}{dx^2}$

SECTION C $(4 \times 3 = 12)$

11. If $y = \tan x + \sec x$, then prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$
12. If $x = a\cos\theta + b\sin\theta, y = a\sin\theta - b\cos\theta$, then show that $\frac{dy}{dx} = -\frac{x}{y}$ and hence show that $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$
13. Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t $\sin^{-1}(2x\sqrt{1-x^2})$.
14. Find $\frac{dy}{dx}$, if $\cos x^y = \cos y^x$

SECTION D $(3 \times 5 = 15)$

15. If $y = \left(x + \sqrt{1+x^2}\right)^n$, then show that $(1+x)^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n^2y$

16.If x = sint, y= sintpt, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$

17.Differentiate $\tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$ with respect to $\cos^{-1}x^2$

Or

Find the value of p and q for which

$$f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x}, \text{if } x < \frac{\pi}{2} \\ p, \text{if } x = \frac{\pi}{2} \\ \frac{q(1-\sin x)}{(\pi-2x)^2}, \text{if } x > \frac{\pi}{2} \end{cases}$$ is continuous at $x=\frac{\pi}{2}$